Odd-even effect in the 2D electron liquid

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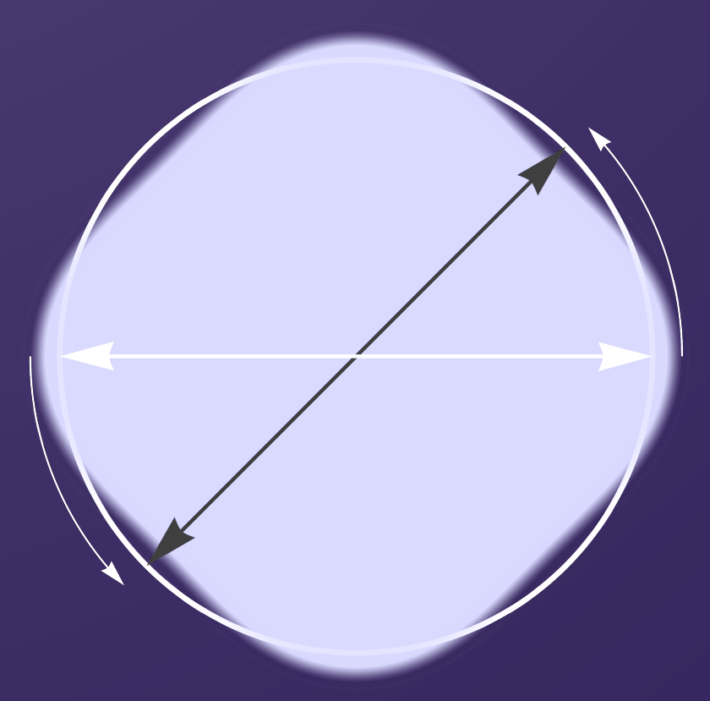
1 Hydrodynamic electrons?

In clean, **two-dimensional materials**, the electrons can reach a **hydrodynamic** transport regime, flowing around obstacles like water.

- The lower carrier densities in 2D means a much lower Fermi temperature than in 3D. The electron-electron decay rate, increasing as $f(T/T_F)$, can therefore become large without the material melting. The electron fluid can then reach local equilibrium and behave hydrodynamically.
- This makes perturbative expansions in T/T_F poor. Hence **there is a need** for an **exact solution** to the governing Fermi liquid equations describing the electrons.

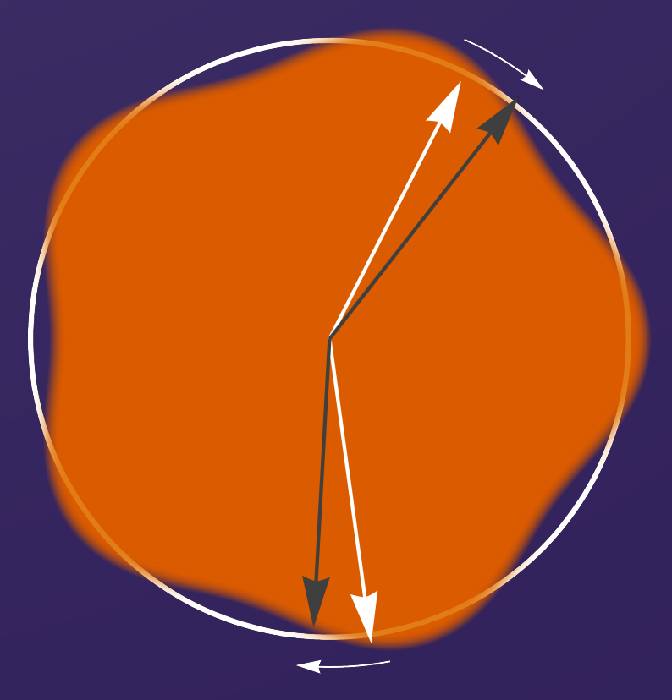
2 Odd-even effect

The kinematics of electrons in 2D are heavily constrained by the scattering phase space set by the Fermi surface. By expanding Fermi surface deformations into angular harmonics, one may classify that:



Even-parity deformations can decay through head-on collisions, leading to a standard Fermi liquid decay rate

$$\gamma_{\text{even}} \sim T^2/T_F$$



Odd-parity deformations instead have to rely on repeated small-angle scattering events assited by the thermal broadening, leading to a **thermally supressed** decay rate

$$\gamma_{\text{odd}} \sim T^4/T_F^3$$

That odd and even Fermi surface deformations relax on parametrically different timescales hints at the existence of a **novel transport regime** in between ballistic and hydrodynamic flow: a "**tomographic**" regime, consisting of hydro modes, in addition to the long-lived odd-parity modes.

3 Methods

We study the linearized kinetic equation describing the electrons semi-classically:

Decay rate

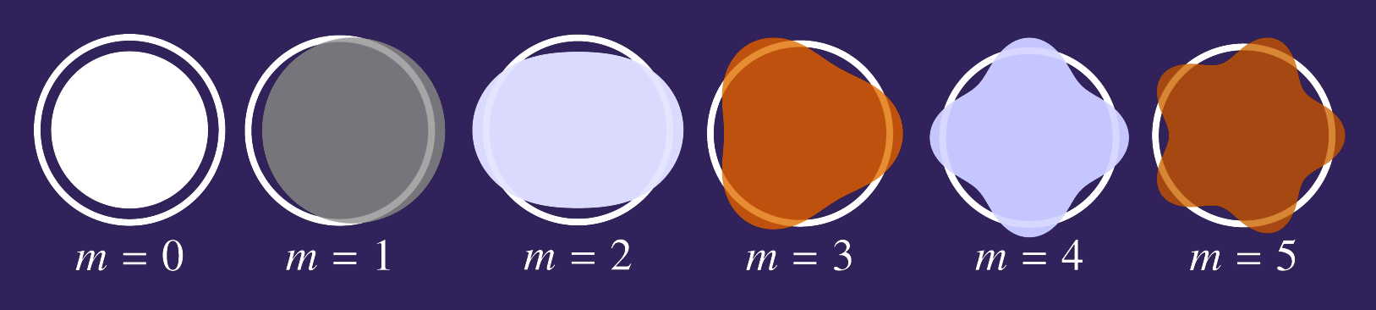
Linearized collision operator

$$\gamma\chi = \mathcal{L}[\chi]$$

Fermi surface deformation

By expanding in angular harmonics and orthogonal polynomials

$$\chi(\boldsymbol{p}) = \sum_{m}\sum_{\ell=1}^{N} c_\ell u_\ell(p) e^{im\theta}$$



We evaluate matrix elements $\langle\chi|\mathcal{L}|\psi\rangle = \frac{m^*\lambda_T^2}{16\pi\hbar^3}\int\frac{\mathrm{d}\boldsymbol{P}\,\mathrm{d}\boldsymbol{q}\,\mathrm{d}\Omega}{(2\pi)^4}|V|^2 F_{121'2}\,[\bar{\chi}(\boldsymbol{p}_1)+\bar{\chi}(\boldsymbol{p}_2)-\bar{\chi}(\boldsymbol{p}_1')-\bar{\chi}(\boldsymbol{p}_2')]\,[\bar{\chi}\to\psi]$ → *Divonne* algorithm (CUBA library, c)

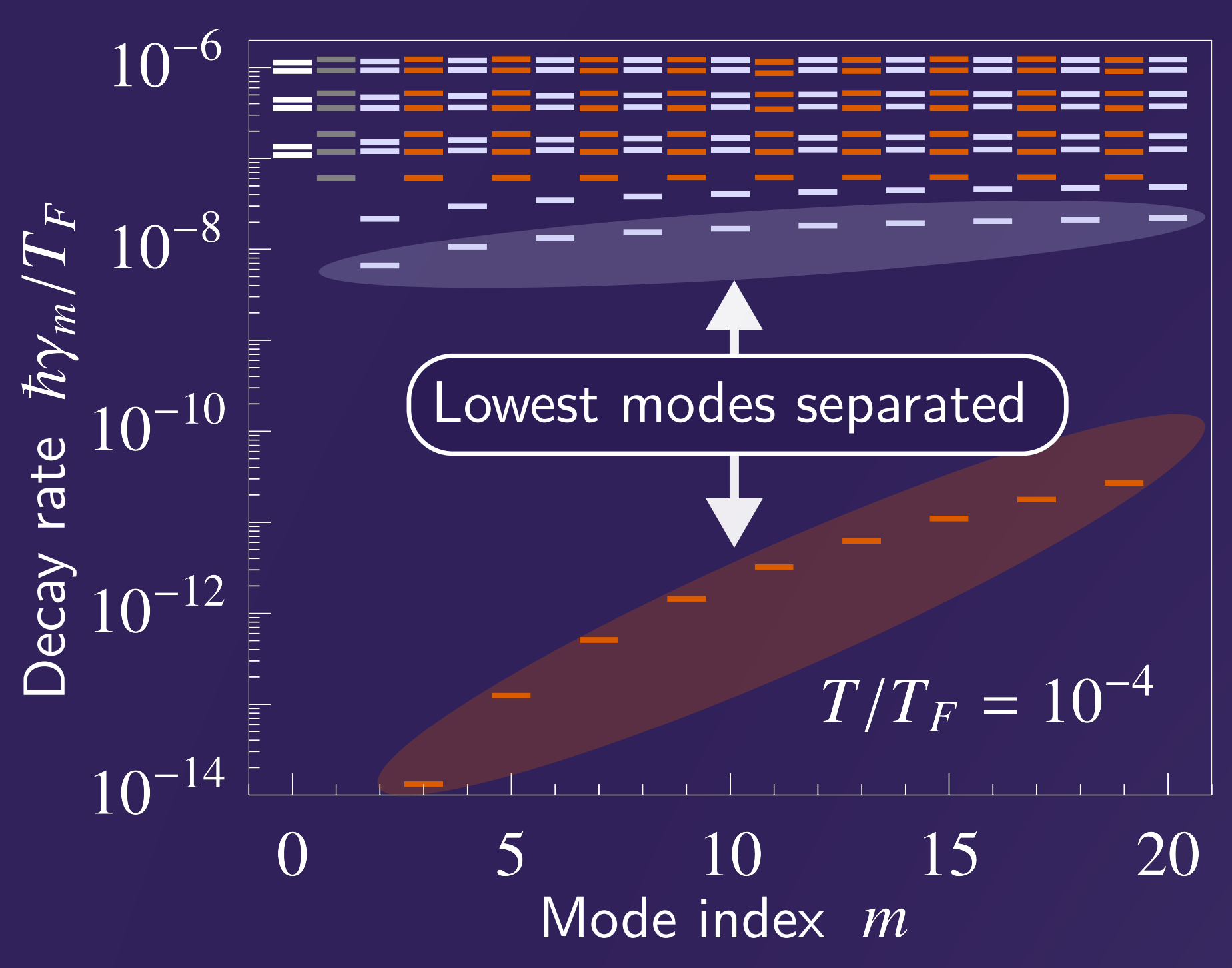
Eigenvalues γ_m

Product of Fermi factors, sharply peaked at low T

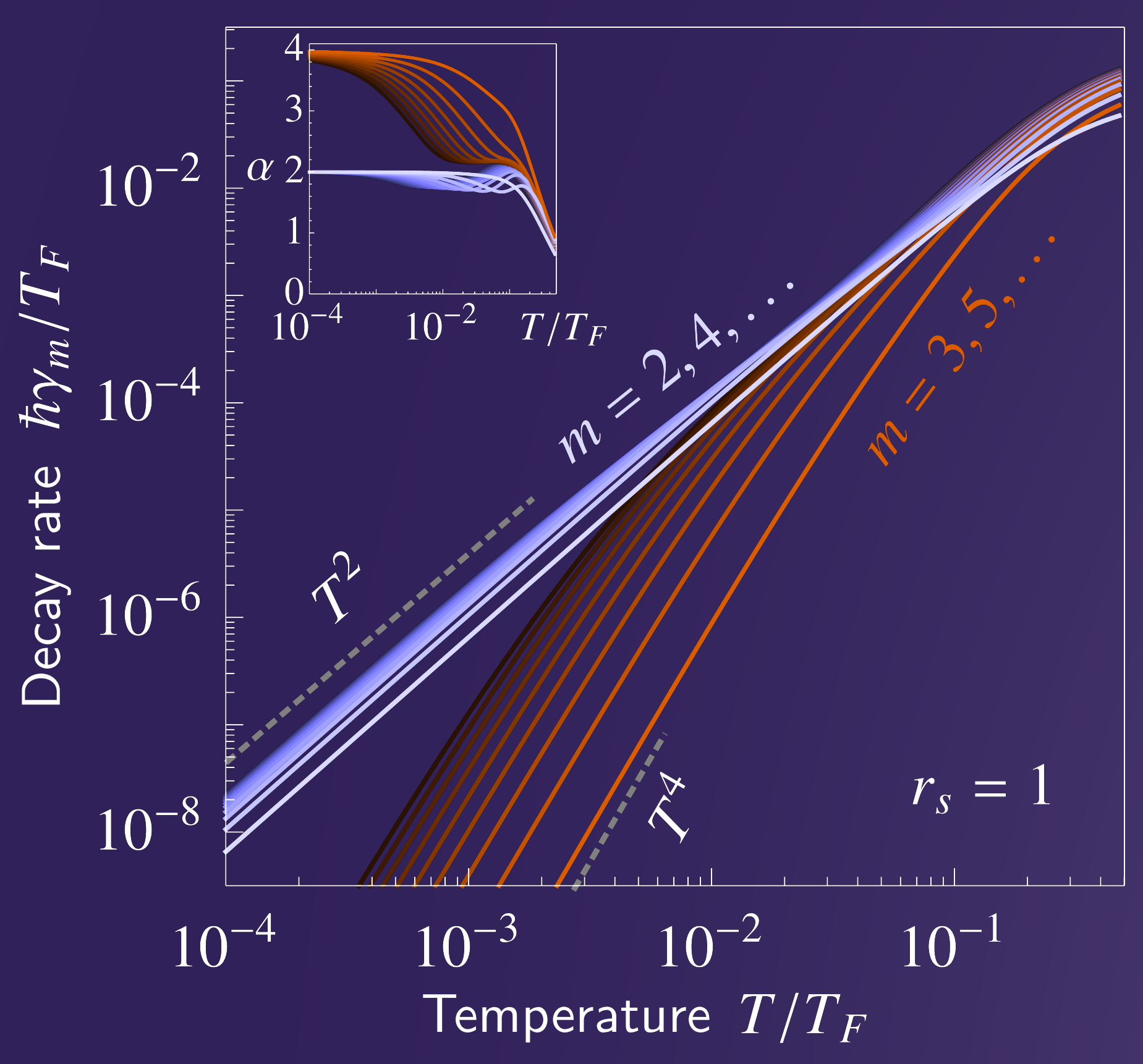
$$V(q;r_s) = \frac{2\pi e^2}{q+\sqrt{2}k_F r_s}$$

Screened Coulomb interaction

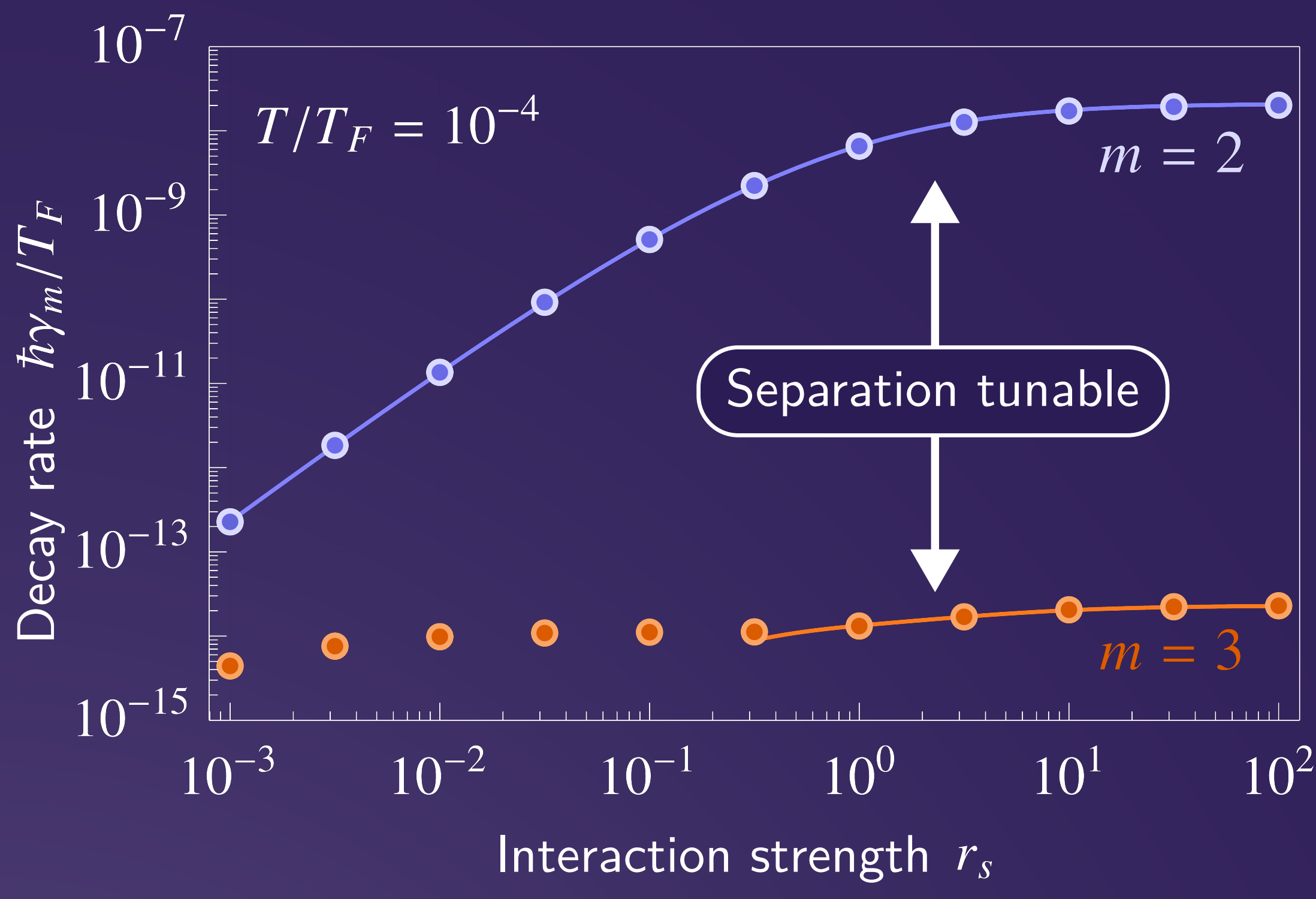
4 Results



We are able to obtain a **complete spectrum** of modes of the collition operator. There exists a set of **long-lived modes**, with a decay rate separated by several orders of magnitude from the rest.



The separation remains up to **experimentally realizable temperatures**. The onset of a regime depends on the angular mode index.



The separation between even and odd parity modes **can be tuned** by the interaction strength between the electrons. Experiments should choose the doping and substrate enviornment to make the effect as large as possible.

5 Conclusions

We have formulated a numerically exact method of describing the interacting 2D electron gas, going beyond the relaxation time approximation or perturbative expansions. This has enabled us to fully characterize a set of long-lived, odd-parity modes that play a role in a tomographic transport regime. The method also allows for the calculation of general linear-response transport coefficients, such as the shear viscosity [2].

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[1] Nilsson, Eric et al. (2024), "Nonequilibrium relaxation and odd-even effect in finite-temperature electron gases", arXiv:2405.03635
[2] Gran, Ulf et al. (2023), "Shear viscosity in interacting two-dimensional Fermi liquids", arXiv:2312.09977

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